



Nanoscale Systems for Optical Quantum Technologies

Grant Agreement No: 712721

Start Date: 1st October 2016 - Duration: 36 months

D3.9 Sensing in hybrid systems

Deliverable:	D3.9
Work package:	WP3 Opto-electrical and opto-mechanical hybrid systems
Task:	3.5 Theoretical analyses, diagnostics and proposal for new experiments
Lead beneficiary:	AU
Type:	Report
Dissemination level:	Public
Due date:	30 September 2019
Actual submission date:	25 September 2019
Author(s):	Klaus Mølmer (AU) Signe Seidelin (CNRS-IN)



This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 712721.

Version history

Version	Date	Author(s)	Description
V1	22/09/2019	K. Mølmer (AU)	First draft
V2	24/09/2019	S. Seidelin (CNRS-IN) K. Mølmer (AU)	Revised version submitted to EU

Copyright Notice

Copyright © 2019 NanOQTech Consortium Partners. All rights reserved. NanOQTech is a Horizon 2020 Project supported by the European Union under grant agreement no. 712721. For more information on the project, its partners, and contributors please see <http://www.nanoqtech.eu/>. You are permitted to copy and distribute verbatim copies of this document, containing this copyright notice, but modifying this document is not allowed.

Disclaimer

The information in this document is provided as is and no guarantee or warranty is given that the information is fit for any particular purpose. The user thereof uses the information at its sole risk and liability.

The document reflects only the authors' views and the Community is not liable for any use that may be made of the information contained therein.

Table of Contents

Deliverable description	4
Introduction and context.....	4
Theoretical approach	4
Conclusion.....	6
Bibliography.....	6

Deliverable description

This deliverable consists in a report on sensing in hybrid systems, and their dependence on damping and dissipation effects. This work being related to task 3.5 "Comparison between FIB samples and thin films for nano-resonators" and milestone 4 "Master equation for the dynamics of rare earth ions coupled to mechanical vibrations" (MS4) and builds on, and extends, the work discussed in D3.8 "Simulation of state dynamics".

Introduction and context

We have in the framework of the NanOQTech project developed theory for force sensing by an optically probed cantilever. The physical system has been described in D3.4 and D3.8, and we here extend the theory in order to benefit from the obtained cold resonator for the goal of using the system as a sensitive force probe. The advantage of this system as a force sensor relies again on the strain-coupling mechanism: when the resonator bends (here, due to an externally applied force) the energy levels of the rare-earth doped ions shift significantly. Moreover, the large stiffness of the resonator, combined with the active feedback, may allow to control the position of the resonator and prevent the resonator from "sticking" to the object generating the force, which is a major issue for nanowires being used as force probes.

Theoretical approach

In brief, the reduction of position and momentum variance increases the sensitivity of the cantilever compared to the state with thermal fluctuations. This is readily accounted for by our formalism by merely incorporating the unknown force in the Gaussian covariance formalism. The starting point is the basic master equations established in D3.4, based on the physical system established in [1] and prepared according to the protocol described in [2]. To estimate the magnitude of the force, using a Bayesian inference approach, we treat it as an unknown variable with an a priori Gaussian distribution. This allows us to describe the combined classical force and quantum position and momentum quadrature operators (F ; X_m ; P_m ; X_{ph} ; P_{ph}), where the subscript m refers to the mechanical oscillator, and ph to the light, by a joint Gaussian distribution. I.e., we can extend our representation of cantilever mean values and covariances to include also the mean value and variance of the probability distribution of the unknown force as well as its covariance with the cantilever and field variables:

$$\begin{pmatrix} F \\ X_m \\ P_m \\ X_{ph} \\ P_{ph} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \omega\tau & 0 & 0 \\ \tau & -\omega\tau & 1 & \kappa_\tau & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \kappa_\tau & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} F \\ X_m \\ P_m \\ X_{ph} \\ P_{ph} \end{pmatrix}$$

The first and third lines of the matrix reflect that the force does not change during the probing, but it induces a change in the momentum. Using the same notation as in the deliverable D3.4 for the cantilever covariance matrix elements, but adding a 0th row and

column for the covariances and variance of the force component, we obtain the coupled equations of motion (equations 1)

$$\begin{aligned}
 \frac{da_{00}}{dt} &= -\eta\kappa^2 a_{01}^2 \\
 \frac{da_{01}}{dt} &= -\eta\kappa^2 a_{01}a_{11} - \frac{\gamma}{2}a_{01} \\
 \frac{da_{02}}{dt} &= -\eta\kappa^2 a_{01}a_{21} - \frac{\gamma}{2}a_{02} + a_{00} - \omega a_{01} \\
 \frac{da_{11}}{dt} &= -\eta\kappa^2 a_{11}^2 + \omega(a_{21} + a_{12}) - \gamma(a_{11} - (2\bar{n} + 1)) \\
 \frac{da_{12}}{dt} &= -\eta\kappa^2 a_{11}a_{12} - \omega(a_{11} - a_{22}) - \gamma a_{12} + a_{10} \\
 \frac{da_{22}}{dt} &= \kappa^2 - \eta\kappa^2 a_{12}a_{21} - \omega(a_{21} + a_{12}) \\
 &\quad - \gamma(a_{22} - (2\bar{n} + 1)) + a_{02}.
 \end{aligned} \tag{1}$$

Simulations

We cannot solve Eqs. (1) analytically, but we can make a qualitative assessment of the solution: for short times, the cantilever position and momentum are not well known, and the probing yields only little information about the force, but then, as the probing gradually reduces the position and momentum variance, the value of the force becomes discernible, and as the position and momentum variance converge to constant values, probing for longer times T improves the variance of the force estimate as $1/T$. These expectations are confirmed by our numerical simulations, as shown in figure 1.

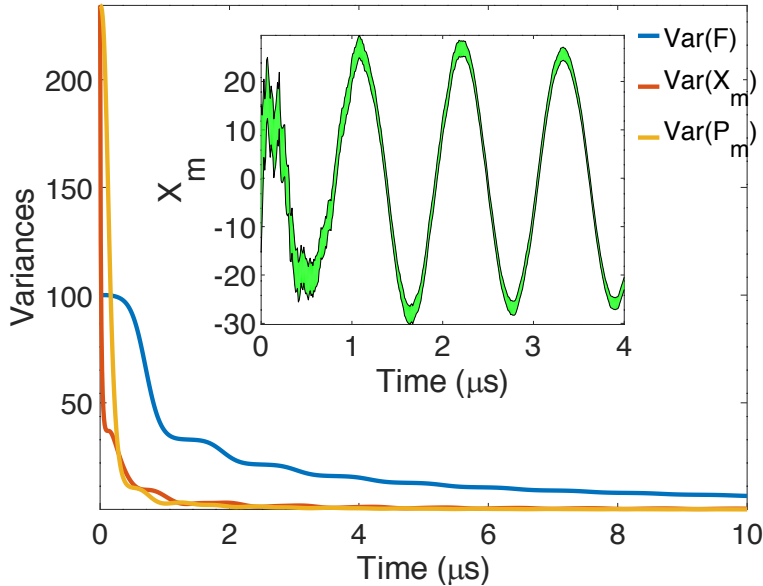


Figure 1 Variances of force, position and momentum for a continuously probed resonator. The inset shows the measured position of the resonator. In this example, the resonator has not been previously cooled, but due to the knowledge of the position of the resonator, this is not required.

Conclusion and outlook

In addition to the generally reduced position, momentum and force uncertainty by continuous probing, as described above, we have also investigated the use of stroboscopic probing to probe only the momentum quadrature, and thus squeeze the oscillator state (this is partly discussed in deliverable D3.7). In toy model calculations, this indeed works very well. For our physical set-up, however, the contact with an environment with a mean thermal excitation of thousands of quanta with even a weak thermal relaxation rate γ is enough to prevent exploration of non-classical regimes of squeezing. Measurement induced reduction of the variance, and stroboscopic probing of the most relevant quadrature improves sensitivity due to the squeezed thermal fluctuations, but further ideas must be introduced to reach the true quantum sensitivity regime.

With or without quantum enhanced sensing (such as the use of a squeezed resonator state), the use of a mechanical resonator should provide an efficient force and inertial sensor, and future work includes extending the theory to account for the detection of time dependent forces on the cantilever. The latter analysis is an ideal case to combine Kalman filter theory and recent ideas for retrodiction with quantum measurement processes.

Bibliography

[1] K. Mølmer, Y. Le Coq and S. Seidelin, *Dispersive coupling between light and a rare-earth ion doped mechanical resonator*, Physical Review A **94**, 053804 (2016).

[2] S. Seidelin, Y. Le Coq and K. Mølmer, *Rapid cooling of a strain-coupled oscillator by an optical phase-shift measurement*, Physical Review A **100**, 013828 (2019)