Nanoscale Systems for Optical Quantum Technologies

Grant Agreement No: 712721
Start Date: 1st October 2016 - Duration: 36 months

D2.3 Conditional dynamics

Deliverable:	D2.3
Work package:	WP2 Spin-atom-photon interfaces
Task:	2.1 Purcell-enhanced readout of a single Eu^{3+} ion
Lead beneficiary:	AU
Type:	Report
Dissemination level:	Public
Due date:	30 September 2018
Actual submission date:	29 September 2018
Author(s):	Klaus Mølmer (AU)

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 712721.

Version history

Version	Date	Author(s)	Description
V1	25/09/2018	K. Mølmer (AU)	Draft
V2	25/09/2018	K. Mølmer (AU) D. Hunger (KIT)	Revised Draft
V3	29/09/2018	K. Mølmer (AU) D. Hunger (KIT) D. Serrano (CNRS-CP) P. Goldner (CNRS-CP)	Submission to EC

Table of Contents

Deliverable description

This deliverable consists in a report on the development of theory for conditional dynamics and quantum computation schemes with ions in cavities.

Introduction and context

This deliverable deals with the establishment of theory that describes detection of optical signals that have interacted with the ions inside a cavity. While density matrix and input-output theory permit calculation of the steady state phase shift and intensity of the reflected signal, a temporal analysis of the signal incorporates the measurement noise and the back action on the emitter system.

As the emitter dynamics is conditioned on the measurement outcomes, it also depends on what quantity is actually measured, e.g., whether the field intensity is measured by photon counting or the field quadrature is measured by homodyne detection. The theory takes on different forms for such different probing scenarios.

While the measurement noise would seem a source of error and prevent the determination of unknown physical parameters in an experiment, quite the opposite is true: due to the measurement back action, the system is continuously quenched, and the resulting transient dynamics is often more sensitive to variations in, e.g., detuning and coupling strengths than the steady state of the system. We have thus developed Bayesian methods that optimally extract physical parameters from stochastic measurement records.

AU has developed theory and a suite of numerical simulation code for the different measurement models and purposes. Direct simulations of the stochastic density matrix equations for simple systems have been supplemented with the development of effective descriptions of more complex systems.

Summary of tools developed in the project

The following lists our main activities, the subsequent section provides examples of their main research outcomes:

Direct numerical simulation of a single two-level ion interacting with a cavity field in the bad cavity regime.

- MATLAB codes for simulation of counting and homodyne (different LO phases) detection.
- MATLAB codes for Bayesian estimation of ion parameters from probing signal (detuning, coupling strength).
- Master equation theory and MATLAB codes have been applied for determination of mean values and spectral correlation functions – to be compared with the simulation results.
- Extension to simulation of a two-ion system, where the probing of one ion informs on the ion-ion interaction strengths and hence location of the second ion (see Figure 1).

Effective description of finite and large numbers of emitters in cavities

- Under the assumption of identical emitters, a collective spin picture is possible and exact simulations have been carried out for few tens of emitters (see Figure 2).
- For many (100 - 10^5) emitters, a cumulant expansion has been applied to the solution of the many emitter dynamics, which includes one- and two-ion observables, but assumes factorization of higher order terms.
- The cumulant expansion has been tested with an assumption of identical emitters and with groups of emitters with up to 10 different transition frequencies.

The effective many-ion codes have been extended to describe spectral measurements, by effectively including a filter cavity as part of the quantum system and determine its mean and conditional excitation (see Figure 3).

Three specific research outcomes

Probing of ion-ion interaction

A two-ion system, with Hamiltonian:

$$\hat{H}_m = \frac{\Omega_s}{2}\hat{\sigma}_x^{(s)} + \frac{\Omega_q}{2}\hat{\sigma}_x^{(q)} - \delta_s|e_s\rangle\langle e_s| + \Delta_m|e_q\rangle\langle e_q| \otimes |e_s\rangle\langle e_s| \quad \text{(Fig. 1 (b))}$$

is monitored with the purpose to determine the ion-ion interaction,

$$\Delta_m = \left(\frac{\epsilon+2}{3\epsilon}\right)^2 \frac{\mu_s\mu_q}{4\pi\epsilon_0 r_m^3}\left[\hat{\mu}_s\cdot\hat{\mu}_q - 3(\hat{\mu}_s\cdot\hat{r}_m)(\hat{\mu}_q\cdot\hat{r}_m)\right]$$

and thus the relative location of the ions to each other. In our numerical example, continuous probing gradually reduces the error in estimation of the location of the second ion among 7 candidate crystal sites, Fig. 1 (a), were the sensitivity is due to the interaction level shift. The two contour plots, Fig. 1 (c,d), show the error as function of the (constant) probe laser detuning and Rabi frequency, respectively. The red curves trace is the optimal set of laser parameters for a given total measurement time (in units of the Purcell decay time).

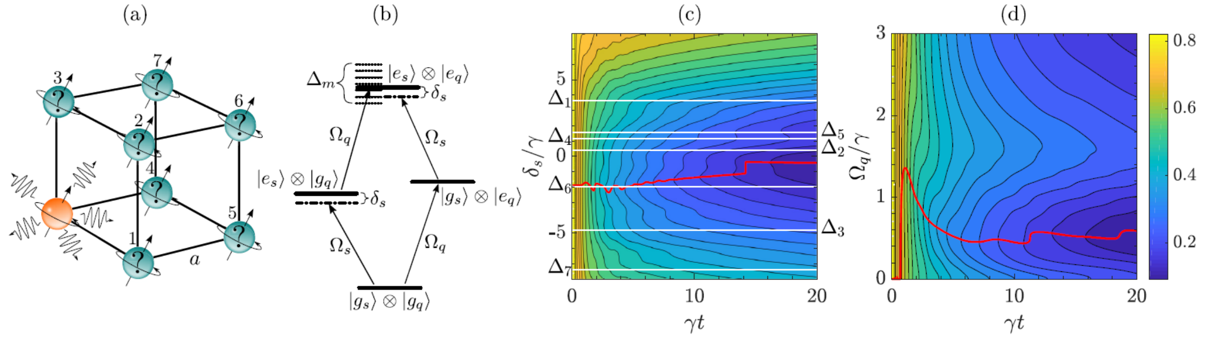

Figure 1. Determination of two-ion configuration by monitoring of cavity reflection signal in the bad cavity limit. (a) candidate configurations, (b) two-ion level scheme, (c,d) error probability as function of time and laser detuning/Rabi frequency [1].

Collective emission of N ions – symmetric state expansion

When N two-level systems are described by identical coherent and damping terms, their quantum density matrix is permutation symmetric. We use this to expand the states on collective spin states |J,M>, with J=N/2, N/2-1, ... ½ or 0, and M ranging from –J to J. We have developed stochastic wave function simulations for this representation, applicable to monitored systems and for efficient simulation of un-observed systems. As an example, we have applied the method to N=50 identical emitters in a bad cavity and show the emitted photon flux as well as the individual trajectories of the ion degrees of freedom in Figure 2.

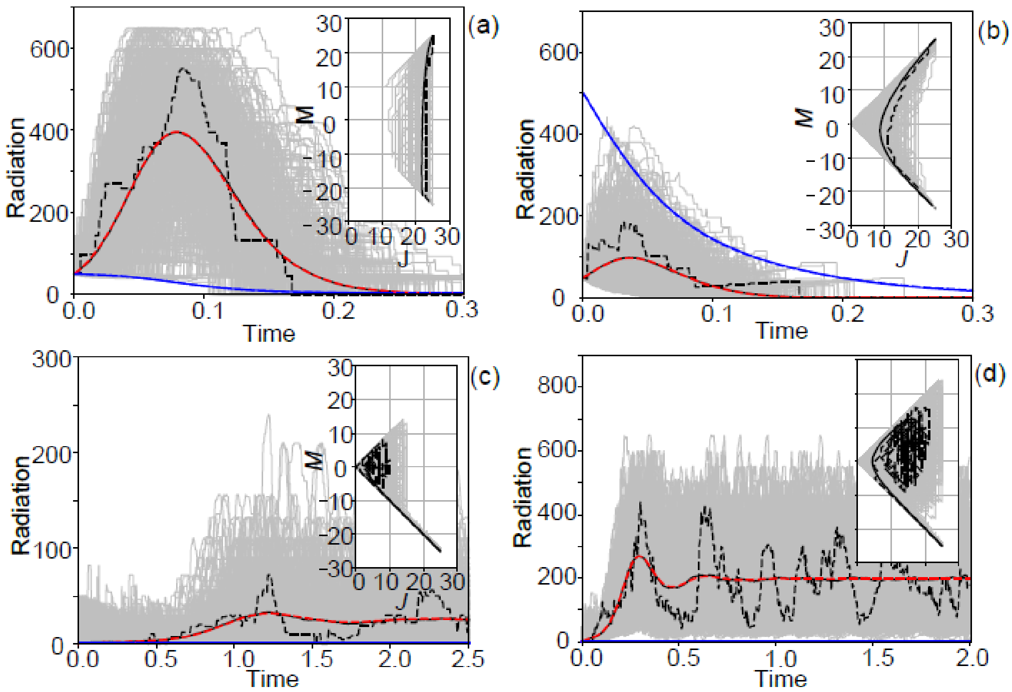

Figure 2. Panel (a) (panel (b)) shows the transient decay of initially excited emitters subject to weak (strong) individual damping. The weak damping case leads to states with high symmetry (large J values in the insert) and strong collective emission behavior, while stronger individual decay leads to exploration of less symmetric states and emission of a weaker signal. In (c) and (d) we explore the dynamics under weak (strong) incoherent driving of the emitters, leading to weak (stronger) excitation (M-values) and steady state emission. Grey curves: 512 trajectories; dashed black: a single random trajectory, red: average values; blue: (atomic emission outside of cavity mode) [2] .

Emission spectrum by ensemble of two-level emitters in a cavity

Using cumulant expansion (truncation of operator products above second order) we can solve problems with an unlimited number of emitters. We have developed general theory and studied its specific application to recent experiments with (identical) strontium atoms which may yield mHz linewidth lasing (see Fig. 3). The programs are now being extended to deal with inhomogeneous broadening of the atomic lines relevant to NanOQTech experimental systems.

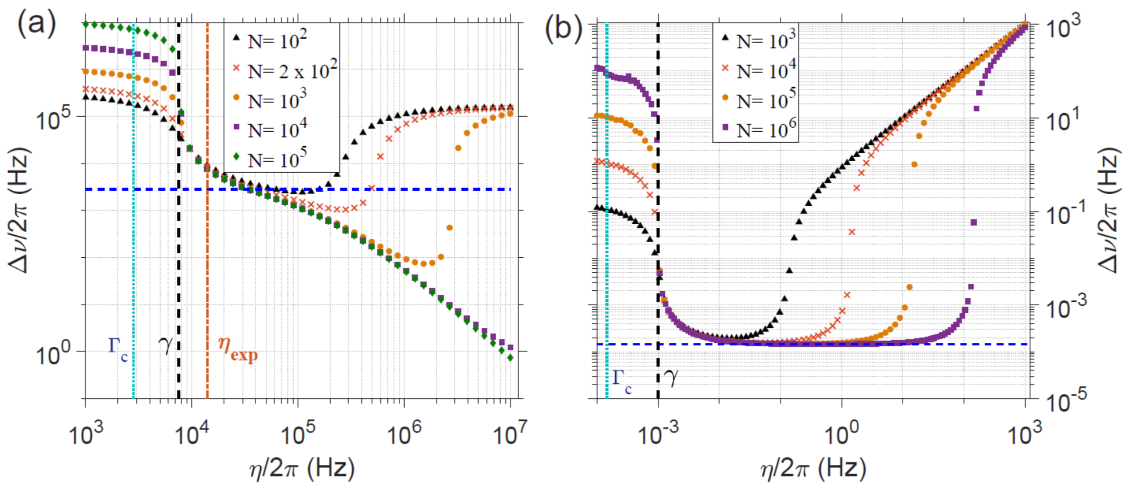

Figure 3. (a) Linewidth of steady state signal from N strontium-88 atoms on the $^3P_1$-$^1S_0$ line as function of the incoherent excitation rate. (b) Same dependence but for the strontium-87 $^3P_0$-$^1S_0$ line. The spectral widths explore superradiance, lasing and cross-over regimes where the frequency is maintained by stimulated emission, collective emission and combinations of the two, respectively [3].

Conclusion

We have in the framework of the NanOQTech project derived exact and effective theories that allow calculation of the properties of fields reflected from the ions in a cavity. The probing of that signal is described by the (stochastic) theory of quantum measurements, and we have produced numerical codes that simulate such dynamics. We have demonstrated the applicability of our methods with examples of direct relevance to the NanOQTech experimental projects on dopant ions and to related activities with neutral atoms in optical lattices.

In succession to the work reported here, we plan to use our theory and numerical methods for interacting ions to benchmark and optimize the ability to apply deterministic and heralded quantum gates between different ions both by the un-monitored and the monitored interaction with probe fields.

Bibliography

[1] Alexander Holm Kiilerich and Klaus Mølmer, *Multistate and multihypothesis discrimination with open quantum systems*, Phys. Rev. A **97**, 052113 (2018). (NanoQTech publication).

[2] Yuan Zhang, Yu-Xiang Zhang and Klaus Mølmer, *Monte-Carlo Simulations of Superradiant Lasing*; https://arxiv.org/abs/1806.02156, (NanoQTech publication, submitted to NJP).

[3] Kamanasish Debnath, Yuan Zhang, and Klaus Mølmer, *Lasing in the superradiant crossover regime*; arXiv:1809.01602 (NanoQTech publication, submitted for publication).