NanOQTech

Nanoscale Systems for Optical Quantum Technologies

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D3.8 Simulation of state dynamics

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Version history

Version	Date	Author(s)	Description
V1	22/09/2019	K. Mølmer (AU)	First draft
V2	25/09/2019	S. Seidelin (CNRS-IN) K. Mølmer (AU)	Revised version submitted to EU

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Deliverable description

This deliverable consists in a report on a simulation tool for state dynamics conditioned on continuous probing in a system of rare earth ions coupled to mechanical vibrations. This work is related to task 3.5 " Theoretical analyses, diagnostics and proposal for new experiments " and milestone 4 "Master equation for the dynamics of rare earth ions coupled to mechanical vibrations" (MS4).

Introduction and context

The simulation of state dynamics of hybrid opto-mechanical systems has in the course of the NanOQTech project been reported in two scientific publications. In [1] (a part of which is described in deliverable D3.4), we have proposed and analysed how a spectral hole in an ion dopant ensemble in a cantilever is distorted under the effect of strain and hence causes a motion dependent phase shift of a laser probe beam. The projected sensitivity of this phase shift allows detection of motion at the quantum level, which is equivalent to an ability to prepare the correspondingly localized quantum states by quantum measurement back action. In a second article recently published [2] (in part corresponding to the work discussed in this report) we have extended the theory and derived equations of evolution for the state of the mechanical oscillator conditioned on the optical detection record, including an active feedback which allows us to cool the resonator to near the quantum ground state.

Theoretical Approach

The theory has its starting point in stochastic master equations and quantum measurement theory, and for the case of oscillator motion and probing with a coherent laser beam, the state is well approximated by a Gaussian Wigner function. Such a phase space distribution is fully characterized by the position and momentum mean values $\langle x \rangle$ and $\langle p \rangle$ and their variances and covariances (a 2-by-2 matrix). We have thus derived an effective theory for these quantities. The mean values depend on the measurement record, while the covariance matrix evolves deterministically under the continuous probing of the system. While the system is not brought deterministically to rest by the interaction, the position and momentum variables become known with progressively reduced uncertainty, equivalent to the low entropy of an oscillator cooled with respect to a (known) moving frame. Moreover, due to the knowledge of these parameters, we can design an active feedback based on the interaction with the probe, which physically cools the resonator’s vibrations. We have investigated the prospect of applying such feedback and hence obtain a deterministic cooling of the cantilever. A realistic protocol with a suitable processing delay, leads to the results shown in the figure with cooling from about 10.000 quanta of excitation (temperature of 400 mK) to about a mere 15 quanta (equivalent to an effective temperature below 1 mK).

Results of simulations

The results of the simulations are shown in figure 1. The physical parameters are: resonator oscillation frequency $\omega = 2\pi \times 1$ MHz, damping rate $\gamma = 2\pi \times 10$ Hz, laser full power 1 mW, initial temperature 400 mK). The sequence simulated in the figure is the following: (1) we start at thermal equilibrium with no probing laser; (2) after 1 µs, we apply the probe laser at half power (feedforward); (3) at 1.5 µs we apply the probe laser at full power; (4) at 2.5 µs we detect the probe laser phase continuously, which progressively localizes the resonator on a sine-wave oscillation with random amplitude and phase; (5) after 10 µs, we apply an active feedback process (which includes a 1-µs delay time) that keeps the resonator near its rest position by modulating the probe laser power.

Our protocol differs from other optomechanical schemes by relying exclusively on the dispersive interaction between the light field and the oscillator motion rather than dissipative cooling forces. Our scheme does not use a cavity, but we note that in the “bad cavity limit” opto-mechanical schemes relying on dispersive effects are generally superior to dissipative schemes. The most crucial aspect of our work, however, is its conditional character. The sensing of the cantilever motion in just a single oscillation period permits a drastic reduction of the position and momentum uncertainty unmatched by attempts to apply unconditional dissipative forces to arrest the motion.

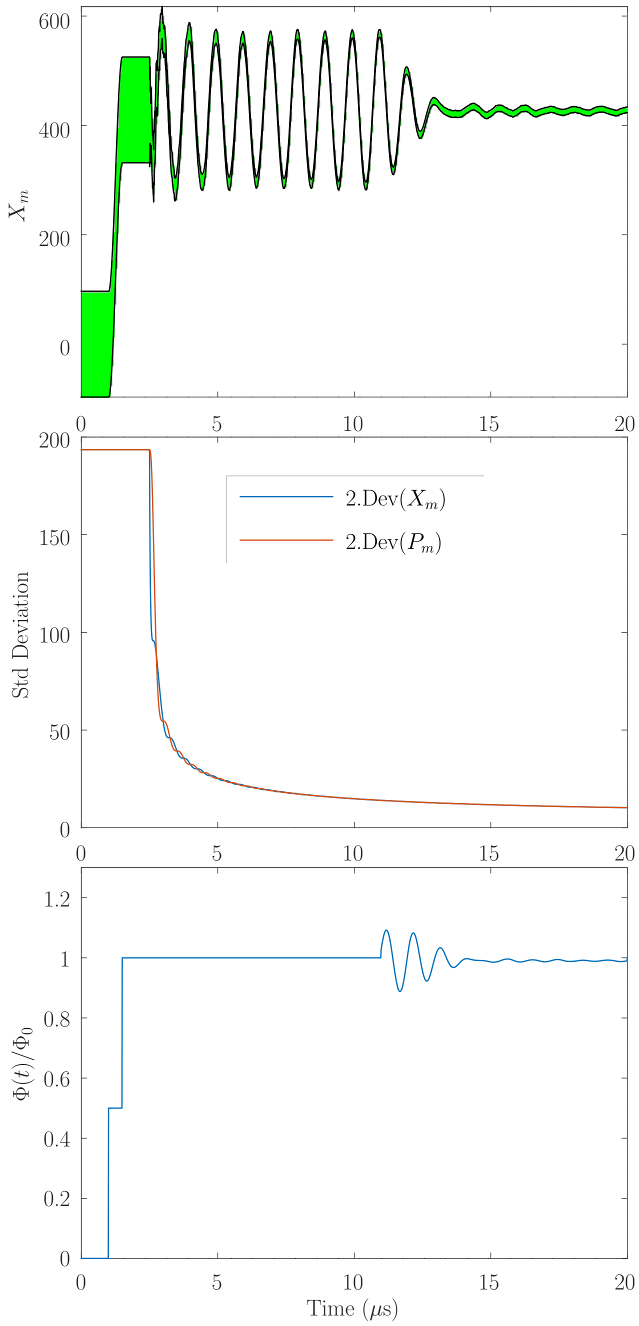

Figure 1 Optical probing and active feedback of resonator and light interaction. Top panel: Mean position of the resonator (the green-shaded area of the curve indicates the uncertainty). Middle panel: Two times the standard deviation (root mean square) of the position and momentum variables. Bottom panel: Variation in the probe laser flux normalized to its final constant value, showing the excitation at half power and the oscillatory feedback modulation, conditioned on the measurement outcome.

Conclusion and outlook

We have derived a Gaussian state formalism that accounts for the evolution of a mechanical oscillator subject to continuous homodyne probing and active feedback cooling. In particular, we have shown that the resonator motion can be arrested by application of a force. In particular, we can use a force which arises from interaction with the probe itself and perform a feedback according to the measurement of the resonator position by varying the intensity of the probe.
In addition to providing an efficient cooling scheme, the theoretical simulation tools outlined in this deliverable also provide a platform for studying the resonator in non-classical states (squeezed states) as discussed in deliverable D3.7.

As mentioned above, our protocol is relying exclusively on the dispersive interaction between the light field and the oscillator motion rather than dissipative cooling forces. Theoretical and experimental studies [3,4] have shown that combining dissipative optomechanical interactions and state estimation by detection of the scattered field supersedes the results of the unconditional dissipative cooling. A more complete picture of the advantages of combining conditional dispersive interactions and unconditional dissipative dynamics to control mechanical motion may rely on separate detailed studies of different systems, and would constitute a natural extension of our work.

Bibliography

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